Int. J. Heat Mass Transfer. Vol. 25, No. 9, pp. 1411–1418, 1982
Printed in Great Britain

0017-9310/82/091411-08 \$03.00/0
Pergamon Press Ltd.

A MULTI-LAYER MODEL FOR MIXING LAYER DEVELOPMENT IN A DOUBLE-DIFFUSIVE THERMOHALINE SYSTEM HEATED FROM BELOW

T. L. BERGMAN, F. P. INCROPERA and R. VISKANTA
Heat Transfer Laboratory, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, U.S.A.

(*Received* 29 *March* 1982)

Abstract—A mathematical model has been developed for determining the dynamic response of a salt-stratified solution to heating from below, and predictions have been compared with experimental results. All major data trends have been corroborated, and good numerical agreement has been obtained for the temperature, salt concentration and thickness of the bottom mixed layer which forms in response to the heating. Underprediction of the time required for thermal effects to penetrate to the top of the solution is attributed to limitations associated with using the energy integral equation to describe conditions in the stable region. Parametric calculations have been performed to determine the effect of bottom heat flux and stabilizing salt gradient on system response.

NOMENCLATURE

a, b, c,	coefficients of temperature profile in the diffusive region;
c_v,	specific heat;
D,	salt diffusion coefficient;
g,	gravitational acceleration;
H,	double-diffusive system height,
k,	thermal conductivity;
m_s,	salt mass fraction;
q,	heat flux;
$q_{b.c}$,	constant value of bottom heat flux reached on tenth day of experiment;
Ri,	Richardson number;
T,	temperature;
t,	time;
t_p,	time required for thermal effects to penetrate to top of system;
t_s,	time required for secondary mixed layers to form in the double-diffusive region;
u_*,	frictional velocity;
z,	vertical coordinate measured from bottom.

Greek symbols

α,	thermal diffusivity;
β_s,	solute expansion coefficient;
β_T,	thermal expansion coefficient;
Δ,	difference of indicated quantity across the interfacial boundary layer;
δ,	mixed and diffusive layer heights;
η,	dimensionless height;
ν,	kinematic viscosity;
ρ,	mass density.

Subscripts

b,	bottom surface or mixed region;
d,	diffusive region;
i,	initial condition;
p,	thermal penetration;
s,	saline;
t,	top surface or mixed region;
T,	thermal;
w,	side wall.

INTRODUCTION

DOUBLE-DIFFUSIVE, thermohaline systems in which one of the diffusing agents (salt) maintains a stable density gradient, while the other agent (heat) produces an unstable gradient are encountered in both environmental and engineering applications. The physical structure of the upper ocean, for example, may be characterized by such a system. In addition, nonconvecting solar ponds utilize the distinctive characteristics of the double-diffusive, thermohaline system in order to provide space heating, process heat, or power generation.

When the local density gradient in such a system becomes unstable, buoyancy driven fluid motion will occur, producing regions of well mixed fluid contiguous to stably stratified fluid. Such is the case when the system is destabilized due to heating from below. The situation is dynamic, with mixed layer expansion or contraction depending on the bottom heating schedule.

The first study of a double-diffusive, thermohaline system heated from below was made by Turner [1], who considered mixed layer growth in a linearly stratified salt solution resulting from vigorous bottom heating. The thickness of the mixed layer which formed at the heating surface was observed to increase as $t^{1/2}$, until a secondary mixed layer formed above the first. The experiments were characterized by the formation of multiple mixed layers, which were separated by diffusive regions, with growth confined to the top layer. Although Poplawsky, Incropera and Viskanta [2] performed similar experiments, the imposed bottom heating was much less vigorous in order to comply with typical solar pond conditions. Accordingly, multiple mixed layer formation did not occur, and events were characterized by the growth of a single bottom mixed layer. As heating effects penetrated throughout the system, however, a top mixed layer began to form and the system consisted of growing top and bottom mixed regions, separated by a stable, double-diffusive region.

Attempts have been made to model the dynamic behavior of a salt-stratified solution heated from below. Applying simple energy and salt balances to the bottom mixed layer, Turner [1] was able to accurately predict the growth of this layer. However, the model assumes negligible heat loss from the bottom mixed layer to the overlying diffusive region and is hence suitable only for rapid mixed layer growth. Huppert and Linden [3] extended Turner's model to predict the development of secondary mixed layers, but the model is, again, suitable only for large bottom heat fluxes. Nielsen and Rabl [4] revised Turner's growth model in an attempt to consider heat losses from the bottom mixed layer. However, the model evaluates the heat loss by assuming a linear, time-independent temperature distribution for the diffusive region, thereby ignoring the actual transient nature of the loss. Meyer [5] developed a model which neglected multiple layer formation and was therefore intended for the smaller bottom heat fluxes associated with solar pond conditions. A key feature of the model was its use of an experimental correlation developed by Turner [6] for enhanced salt transport across a thin diffusive region. Use of the correlation is questionable, however, since it was developed for a diffusive region which separates two mixed layers and not for an interfacial boundary layer separating a mixed layer from a diffusive layer, which is the condition associated with the system being modeled. Moreover, although Turner [6] concluded that the transport of heat, as well as the transport of salt, is enhanced, Meyer arbitrarily chose to neglect the enhancement of heat transfer through the interfacial boundary layer.

Although several models have been developed to describe double-diffusive, thermohaline behavior, they are typically subject to severe limitations, particularly in their ability to describe transient events. It is therefore the objective of this work to develop a model for double-diffusive, thermohaline systems heated from below which is applicable to a broad range of conditions and can be used to describe transient phenomena. Specifically the model seeks to accurately describe bottom and top mixed layer behavior for systems subjected to the low levels of bottom heating characteristic of solar ponds. The model also seeks to predict temperature and salt concentration profiles within the system and to predict the formation of secondary mixed layers under conditions conducive to such formation. Comparisons will be made between the model predictions and experimental results obtained under laboratory conditions.

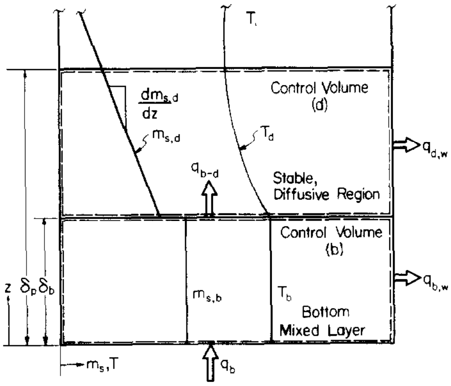

FIG. 1. Two-layer model for response of salt-stratified solution to bottom heating ($t < t_p$).

MATHEMATICAL MODEL

The initial response of a double-diffusive, thermohaline solution to heating from below is characterized by a growing bottom mixed layer whose temperature increases as it advances into the stable, diffusive region. In addition the layer acts as a heat source for the diffusive region, and a destabilizing temperature distribution begins to develop in this region. At some time t_p, the effects of bottom heating penetrate to the top surface of the system, and a top mixed layer begins to develop due to heat loss from the surface to the surroundings. Hence for $t < t_p$, the system may be described by a two-layer model, while for $t \geqslant t_p$ a three-layer model must be used.

Two-layer model

A 1-dim., two-layer model is developed subject to the following assumptions:

(i) The interfacial boundary layer separating the bottom mixed layer from the diffusive region is thin, allowing the internal energy of the layer to be neglected.

(ii) Similarly, the thickness of the thermal boundary layer which forms on the bottom heating surface is negligible, allowing its internal energy to be neglected.

(iii) The transport of salt and the temperature difference across the interfacial boundary layer are negligible.

(iv) The temperature and salt concentration within the mixed layer are uniform.

(v) Terms involving the time rate of change of thermophysical properties are negligible.

(vi) The system is initially characterized by a uniform temperature corresponding to that of the surroundings and a uniform salt concentration gradient.

The validity of assumptions (i)–(iv) has been confirmed experimentally [2, 7], and the validity of assumption (v) has been verified from calculations involving inclusion of the terms in the model. Subject to the foregoing assumptions, the model equations may be obtained by applying conservation principles to control volumes associated with the two-layer system of Fig. 1. One control volume is drawn about the bottom mixed layer and is of height δ_b, which increases with increasing time. The second control volume is drawn about that portion of the diffusive region which extends from the top of the bottom mixed layer height, $z = \delta_b$, to the thermal penetration height, $z = \delta_p$, which also increases with time. To render the model applicable to finite laboratory test cells, provision is also made for the inclusion of side wall heat losses from the mixed and diffusive regions.

Applying conservation of energy to control volume (*b*), it follows that

$$\frac{\mathrm{d}T_b}{\mathrm{d}t} = \frac{q_b - q_{b,w} + k_b \left.\dfrac{\partial T_d}{\partial z}\right|_{z=\delta_b}}{\rho_b c_{v,b} \delta_b} \tag{1}$$

where q_b is the applied bottom heat flux and $q_{b,w}$ is the side wall heat loss per unit bottom surface area. The third term on the RHS of the equation accounts for heat loss to the diffusive region, $q_{b\text{-}d}$, which is evaluated by using the Fourier–Biot law with the temperature distribution for the region. Applying conservation of species to control volume (*b*), it also follows that

$$m_{s,b}(t) = m_{s,b,i} + \frac{\mathrm{d}m_{s,d}}{\mathrm{d}z}\frac{\delta_b(t)}{2}. \tag{2}$$

The salt concentration difference across the interfacial boundary layer may then be expressed as

$$\Delta m_{s,b\text{-}d}(t) = -\frac{\mathrm{d}m_{s,d}}{\mathrm{d}z}\frac{\delta_b(t)}{2} \tag{3}$$

where

$$\Delta m_{s,b\text{-}d}(t) \equiv m_{s,b}(t) - m_{s,d}\big|_{z=\delta_b}. \tag{4}$$

Since heat transfer in the diffusive region occurs exclusively by conduction, the integral form of the heat equation [8] may be used to determine the thermal penetration height. Introducing the following dimensionless form of the space coordinate:

$$\eta = \frac{z - \delta_b}{\delta_p - \delta_b} \tag{5}$$

the integrated heat equation may be expressed as

$$\frac{\mathrm{d}T_b}{\mathrm{d}t}\left\{\int_0^1 (\delta_p - \delta_b)[T_d(\eta, t)]\mathrm{d}\eta\right\} - T_i\frac{\mathrm{d}\delta_p}{\mathrm{d}t} + T_b\frac{\mathrm{d}\delta_b}{\mathrm{d}t} + \frac{q_{w,d}}{\rho_d c_{v,d}} = -\alpha_d \left.\frac{\partial T_d}{\partial \eta}\right|_{\eta=0} \Big/ (\delta_p - \delta_b). \tag{6}$$

Use of this form of the heat equation requires the specification of a characteristic temperature profile, and measurements [2] suggest that conditions in the diffusive region are well characterized by a second-order profile of the form

$$T_d(\eta, t) = a(t) + b(t)\eta + c(t)\eta^2 \tag{7}$$

where the coefficients are determined from the conditions

$$T_d(0, t) = T_b(t),$$

$$T_d(1, t) = T_i,$$

$$\frac{\partial T_d}{\partial \eta}(1, t) = 0.$$

The first equation is a thermal equilibrium requirement for the interface between the mixed and diffusive layers, and the remaining equations are inherent conditions for the thermal penetration height. Evaluating the coefficients, equation (7) may be used with equations (1) and (6) to obtain

$$\frac{\mathrm{d}T_b}{\mathrm{d}t} = \frac{q_b - q_{b,w} + 2k_b(T_i - T_b)/(\delta_p - \delta_b)}{(\rho_b c_{v,b} \delta_b)} \tag{8}$$

and

$$\frac{\mathrm{d}\delta_p}{\mathrm{d}t} = \left[\frac{-6\alpha_d(T_i - T_b)}{(\delta_p - \delta_b)} - \frac{\mathrm{d}T_b}{\mathrm{d}t}(\delta_p - \delta_b) - 2\frac{\mathrm{d}\delta_b}{\mathrm{d}t}(T_b - T_i) - 3\frac{q_{w,d}}{\rho_d c_{v,d}}\right] \Big/ (T_b - T_i). \tag{9}$$

The growth of the bottom mixed layer in a stratified system which is heated from below has been determined to be directly proportional to a frictional velocity associated with turbulence in the mixed layer, $u_{*,b}$, and inversely proportional to the overall Richardson number for the layer [9]. The specific correlation may be expressed as

$$\frac{\mathrm{d}\delta_b}{\mathrm{d}t} = 0.20\, u_{*,b} Ri_b^{-1} \tag{10}$$

where

$$Ri_b = \frac{g \left.\dfrac{\partial \rho}{\partial m_s}\right|_T \Delta m_{s,b\text{-}d}\delta_b}{\rho_b u_{*,b}^2} \tag{11}$$

and

$$u_{*,b} = \left[g\beta_T \frac{q_b}{\rho_b c_{v,b}}\delta_b\right]^{1/3}. \tag{12}$$

Combining equations (10)–(12), it follows that

$$\frac{\mathrm{d}\delta_b}{\mathrm{d}t} = 0.20\left(\frac{\beta_T q_b}{c_{v,b} \left.\dfrac{\partial \rho}{\partial m_s}\right|_T \Delta m_{s,b\text{-}d}}\right). \tag{13}$$

This expression has been shown [10] to accurately correlate the mixed layer growth data of Poplawsky *et al.* [2].

Equations (8), (9) and (13) are coupled and may be solved simultaneously for the time dependence of T_b, δ_b, and δ_p. Equation (7) may then be used with equation (5) to determine the temperature profile in the diffusive region as a function of time. The system of coupled differential equations was solved using a Bashforth–Moulton fourth order predictor–corrector technique [11], the starting values of which were determined by a fourth order Runge–Kutta technique [12]. Equations (8), (9) and (13) were rewritten in finite-difference form in order to determine initial

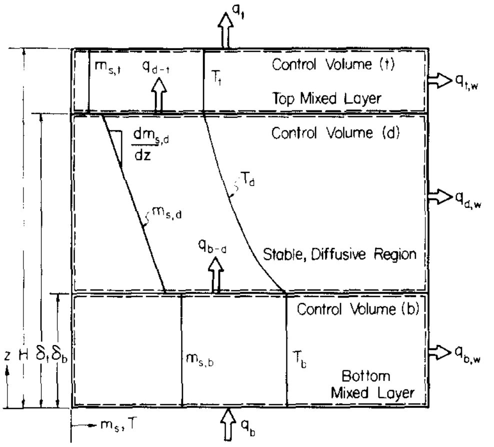

FIG. 2. Three-layer model for response of salt-stratified solution to bottom heating ($t \geqslant t_p$).

perturbation values for the dependent variables [10]. Property variations were considered using curves fitted to existing saline water data [13]. The system of equations was solved until δ_p reached the height of the thermohaline system, $\delta_p = H$, at time $t = t_p$.

Three-layer model

A three-layer model was used for times greater than t_p, when the system is characterized by growing top and bottom mixed layers (Fig. 2). Development of the bottom mixed layer continues to be governed by the bottom heat addition, q_b, while development of the top mixed layer is determined by heat loss from the top of the system, q_t. Depending on the nature of the surroundings to which the surface is exposed, this loss may be due to convective, evaporative and/or radiative transfer processes. Uniform temperature and salt concentration are assumed for the top mixed region, and the first and third assumptions of the two-layer model are applied to the interfacial boundary layer separating the top mixed region from the diffusive region. The system initial conditions correspond to those predicted by the two-layer model for $t = t_p$.

Applying an energy balance to a control volume (t), which extends from the surface at $z = H$ to the bottom of the mixed layer at $z = \delta_t(t)$, the time rate of change of the top layer temperature may be expressed as

$$\frac{dT_t}{dt} = \frac{-q_t - q_{t,w} - k_t \left.\frac{\partial T_d}{\partial z}\right|_{z=\delta_t}}{\rho_t c_{v,t}(H - \delta_t)}. \tag{14}$$

Applying conservation of species to the control volume, the salt mass friction in the top layer may be expressed as

$$m_{s,t}(t) = m_{s,t}(t_p) - \frac{dm_{s,d}}{dz} \frac{[H - \delta_t(t)]}{2} \tag{15}$$

in which case

$$\Delta m_{s,d-t} = -\frac{dm_{s,d}}{dz} \frac{[H - \delta_t(t)]}{2} \tag{16}$$

where

$$\Delta m_{s,d-t} \equiv m_{s,d}\big|_{z=\delta_t} - m_{s,t}(t). \tag{17}$$

Use of the foregoing equations requires knowledge of the mixed layer heights, δ_b and δ_t. The variation of the bottom mixed layer height with time may again be obtained from the Richardson number criterion proposed by Deardoff *et al.* [9], equation (13), and the time dependence of the top mixed layer height may be obtained by extending this criterion to the top layer. In particular, paralleling the development of equations (10) and (13), it follows that

$$\frac{d\delta_t}{dt} = -0.20 \left(\frac{\beta_t q_t}{c_{v,t} \left.\frac{\partial \rho}{\partial m_s}\right|_T \Delta m_{s,d-t}} \right). \tag{18}$$

The integral form of the heat equation may again be used to describe conditions in the diffusive region, where the dimensionless vertical coordinate is now defined as

$$\eta = \frac{z - \delta_b}{\delta_t - \delta_b}. \tag{19}$$

The equation is of the form

$$\frac{d}{dt}\left[\int_0^1 (\delta_t - \delta_b) T_d(\eta, t) d\eta\right] - T_t \frac{d\delta_t}{dt} + T_b \frac{d\delta_b}{dt} + \frac{q_{w,d}}{\rho_d c_{v,d}} = \alpha_d(\delta_t - \delta_b)\left(\left.\frac{\partial T_d}{\partial \eta}\right|_{\eta=1} - \left.\frac{\partial T_d}{\partial \eta}\right|_{\eta=0}\right) \tag{20}$$

where the second-order temperature profile given by equation (7) may again be used. Two of the conditions needed to determine the coefficients may be obtained from the interface thermal equilibrium requirements

$$T_d(0, t) = T_b(t),$$
$$T_d(1, t) = T_t(t).$$

The third condition is obtained from equation (20), which is solved for $\partial T_d/\partial \eta|_{\eta=0}$ and equated to $b(t)$. Completing the evaluation of the coefficients, equations (14) and (20) may be expressed as

$$\frac{dT_t}{dt} = \frac{-q_t - q_{w,t} - k_t \left(2\frac{T_t - T_b}{\delta_t - \delta_b} - \left.\frac{\partial T}{\partial z}\right|_{z=\delta_b}\right)}{\rho_t c_{v,t}(H - \delta_t)} \tag{21}$$

and

$$\frac{d\left(\left.\frac{\partial T_d}{\partial z}\right|_{z=\delta_b}\right)}{dt} = 2\left[6\alpha_d\left(\frac{T_t - T_b}{\delta_t - \delta_b} - \left.\frac{\partial T_d}{\partial z}\right|_{z=\delta_b}\right) - T_b\left(\frac{d\delta_b}{dt} - 2\frac{d\delta_t}{dt}\right) - T_t\left(\frac{d\delta_t}{dt} - \frac{d\delta_b}{dt}\right) - (\delta_t - \delta_b)\right.$$

$$\times \frac{\mathrm{d}T_{\mathrm{t}}}{\mathrm{d}t} - 2(\delta_{\mathrm{t}} - \delta_{\mathrm{b}})\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}t} - \frac{\partial T_{\mathrm{d}}}{\partial z}\bigg|_{z=\delta_{\mathrm{b}}} (\delta_{\mathrm{t}} - \delta_{\mathrm{b}})$$

$$\left(\frac{\mathrm{d}\delta_{\mathrm{t}}}{\mathrm{d}t} - \frac{\mathrm{d}\delta_{\mathrm{p}}}{\mathrm{d}t}\right) - \frac{3q_{\mathrm{w,d}}}{\rho_{\mathrm{d}}c_{\mathrm{v,d}}}\Bigg]\Bigg/(\delta_{\mathrm{t}} - \delta_{\mathrm{b}})^2. \qquad (22)$$

Equations (1), (13), (18), (21) and (22) are coupled and may be solved simultaneously for the time dependence of T_{b}, δ_{b}, $\left.\frac{\partial T_{\mathrm{d}}}{\partial z}\right|_{z=\delta_{\mathrm{b}}}$, T_{t} and δ_{t}. The solution was effected by using the predictor–corrector technique previously described. Equations (7) and (19) could then be used to determine the temperature distribution in the diffusive region.

An additional feature of the behavior of salt-stratified solutions heated from below relates to the formation of secondary mixed layers in the diffusive region [1]. Since the formation of such layers would negate the applicability of the two and three-layer models, it is desirable to develop a scheme for predicting their onset. Secondary layers develop from instabilities which originate when the destabilizing effect of the local temperature gradient exceeds the stabilizing effect of the local concentration gradient. Since the maximum temperature gradient exists at the bottom of the diffusive region ($z = \delta_{\mathrm{b}}$), it is this gradient which should be used to check for the occurrence of instabilities. Accordingly, using the criterion of Weinberger [14], it follows that instabilities will occur when the following inequality is satisfied

$$\left.\frac{\partial T_{\mathrm{d}}}{\partial z}\right|_{z=\delta_{\mathrm{b}}} \beta_{\mathrm{T}}(\nu - \alpha) - \frac{\mathrm{d}m_{\mathrm{s,d}}}{\mathrm{d}z}\beta_{\mathrm{s}}(\nu + D) \geqslant 0. \qquad (23)$$

This stability criterion was tested at each time step during the calculations, and when it was satisfied the calculations were terminated.

RESULTS

To test the validity of the foregoing models, predictions must be compared with data obtained under controlled laboratory conditions. The data should include the temporal variation of top and bottom mixed layer heights and temperatures, as well as temporal variation of the diffusive layer temperature distribution. Such data have been obtained by Poplawsky *et al.* [2] for a test cell of height $H = 0.68$ m and salt gradients ($\mathrm{d}m_{\mathrm{s}}/\mathrm{d}z$) of -10 and -38%/m. In each experiment the bottom heating was gradually increased over a 10 day period to a value, $q_{\mathrm{b,c}}$, which was held constant for the remainder of the experiment. Although this value could not be determined exactly, it is known to be in the range from 83 to 125 W m^{-2} [10]. Accordingly, for a given experiment, three sets of calculations were performed in which the heat flux was assumed to increase linearly from $q_{\mathrm{b}} = 0$ at $t = 0$ to $q_{\mathrm{b}} = 83$, 105 or 125 W m^{-2} at $t = 10$ days. Since the test cell in which the experiments were performed was covered, the heat loss from the top surface, q_{t}, was computed from surface and cover energy balances which accounted for transfer due to radiation and free convection. In addition, side wall heat losses were computed from expressions which related these losses to the heights and temperatures of the different layers.

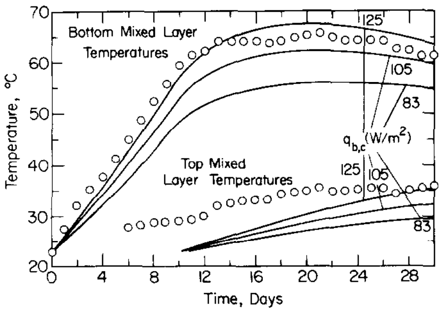

FIG. 3. Comparison of predicted and measured mixed layer temperatures in a salt-stratified solution ($\mathrm{d}m_{\mathrm{s}}/\mathrm{d}z = -38\%$/m) heated from below.

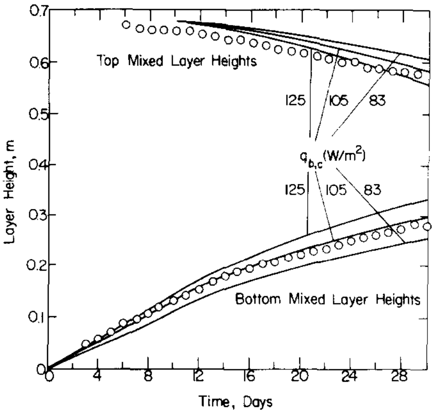

FIG. 4. Comparison of predicted and measured mixed layer heights in a salt-stratified solution ($\mathrm{d}m_{\mathrm{s}}/\mathrm{d}z = -38\%$/m) heated from below.

Comparisons between predictions and data obtained for a salt gradient of $\mathrm{d}m_{\mathrm{s}}/\mathrm{d}z = -38\%$/m are shown in Figs. 3 and 4 for the mixed layer temperatures and heights, respectively. The agreement is generally good, and the best results appear to be associated with a heat flux between 105 and 125 W m^{-2}. The large discrepancy between predicted and measured temperatures for the top mixed layer is not an indication of model inaccuracies but is, instead, due to the existence of thermal stratification at the start of the experiment (initial temperatures at the bottom and top of the tank were 22.5°C and 27.5°C, respectively). Since the model calculations assumed the entire system to be initially at 22.5°C, it would be unreasonable to expect agreement between the predicted and measured top layer temperatures.

A more significant discrepancy is associated with the thermal penetration time, t_{p}. Predicted values ranged from 10 to 11 1/2 days, while the formation of a top mixed layer was observed to occur on day 6 of the experiment. The discrepancy is likely to be due to the use of the integral form of the heat equation and the fact that the thermal penetration time is known to

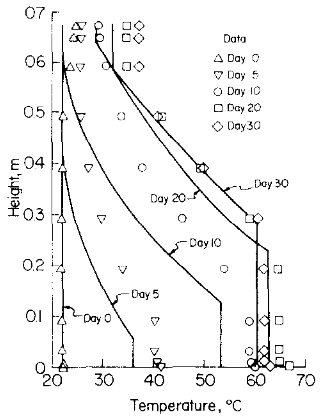

FIG. 5. Measured temperature distributions in a salt-stratified solution ($dm_s/dz = -38\%/m$) and comparison with predictions for $q_{b,c} = 105\ W\ m^{-2}$.

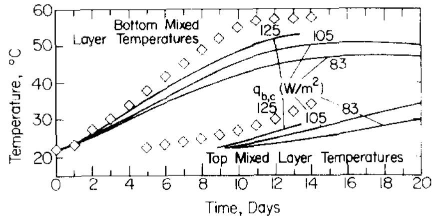

FIG. 7. Comparison of predicted and measured mixed layer temperature in a salt-stratified solution ($dm_s/dz = -10.3\%/m$) heated from below.

depend strongly on the form of the assumed temperature profile [15]. If the predicted top layer temperatures are shifted to the left by 4–5 days (to compensate for inaccuracies associated with use of the integral form of the heat equation) and shifted upwards by 5°C (to compensate for the effect of stratification in the experimental system), good agreement would be obtained between the predicted and measured results. The same may be said for the top mixed layer height of Fig. 4. That is, if the predicted results are shifted to the left by an amount corresponding to the difference between the predicted and actual values of t_p, excellent agreement is obtained for the $q_{b,c} = 105\ W\ m^{-2}$ predictions.

A comparison of measured temperature distributions with predictions obtained for $q_{b,c} = 105\ W\ m^{-2}$ is shown in Fig. 5. As for the results of Fig. 3, calculated temperatures throughout the system are less than the measured temperatures, although, in general, the major trends are predicted and numerical agreement is very good in the period from 20 to 30 days. The development of a linear distribution in the diffusive region is predicted, along with the reduction in bottom mixed layer temperature. This reduction is a consequence of the fact that, for control volume (*b*), heat transfer to the diffusive region and wall losses eventually exceed the rate of bottom heat addition.

As shown in Fig. 6, the bottom mixed layer salt concentration is well predicted by the model, which, from equation (2), is a direct consequence of the ability to predict the mixed layer height. Similarly, from equation (15), differences between predicted and measured concentrations for the top mixed layer may be attributed to difficulties associated with predicting the layer height. If the predicted thermal penetration time is shifted downward by 4 days, however, the agreement would be excellent.

The foregoing experiment was terminated after 30 days, with no sign of diffusive layer instabilities. This was not the case for a second experiment performed using a much smaller salt gradient of −10.3%/m. With a reduction in the gradient, the density difference across the interfacial boundary layers diminishes and the layers offer less resistance to entrainment due to the bombardment of thermal plumes. Accordingly, growth of the primary mixed layers is enhanced, along with the prospects of secondary mixed layer formation in the diffusive region. The formation of such a layer was observed on the 14th day of the experiment and was soon followed by the existence of complete mixing throughout the system.

The second experiment provides a more demanding test of the two and three-layer models, and comparisons are provided in Figs. 7 and 8 for the mixed layer temperatures and heights, respectively. The predictions were terminated when the stability criterion given by equation (23) was satisfied. The predicted

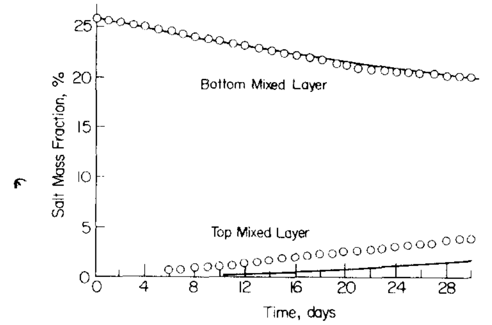

FIG. 6. Measured mixed layer salt concentrations for $dm_s/dz = -38\%/m$ and comparison with predictions for $q_{b,c} = 105\ W\ m^{-2}$.

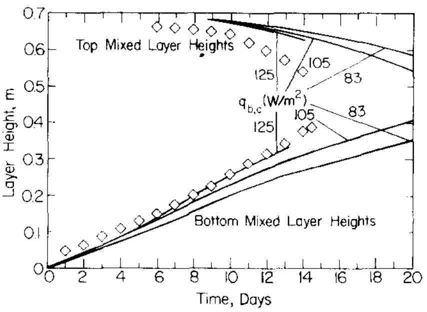

FIG. 8. Comparison of predicted and measured mixed layer heights in a salt-stratified solution ($dm_s/dz = -10.3\%/m$) heated from below.

temperatures and heights are consistently lower than the measured results, and the time at which the top mixed layer begins to form is again overpredicted by approximately 4 days. However, the major trends are corroborated, particularly for the prescribed heat flux of 125 W m^{-2}. The time, t_s, at which secondary layer formation is predicted for this flux ($\approx$13 1/2 days) is virtually identical to that observed experimentally. Note that any effort to improve agreement for T_b by increasing $q_{b,c}$ above 125 W m^{-2} would come at the expense of poorer agreement for t_s.

Comparisons made for the vertical temperature distribution and the mixed layer salt concentrations revealed results similar to those of Figs. 5 and 6, although a linear temperature distribution in the diffusive region and a reduction in the bottom mixed layer temperature were not reached. Failure to achieve these conditions was due to the early formation of secondary mixed layers.

The three-layer model was also used in parametric calculations for which the bottom heat flux and salt gradient were systematically varied from 80 to 120 W m^{-2} and from -6.5 to 26%/m, respectively. For these calculations, side wall losses were ignored and the top surface was assumed to exchange heat directly with ambient air. Accordingly, an evaporative heat loss term was included with radiative and convective terms in evaluating the surface heat loss, q_t [10]. The calculations were performed for a wind speed of 8 km h^{-1}, a system depth of 1 m, and an ambient air temperature and relative humidity of 25°C and 100%, respectively.

Consistent with the foregoing results, the parametric calculations indicate that, with increasing bottom heat flux, the mixed layer temperatures and thicknesses increase, while the time required for thermal effects to penetrate to the top surface, t_p, and for secondary mixed layers to form, t_s, decreases. With increasing salt concentration gradient, the temperature of the bottom mixed layer increases, while that of the top layer decreases, and the thickness of both layers decreases. In addition, both t_p and t_s increase with increasing salt gradient.

Results obtained for the maximum bottom layer temperature and the number of days of stable operation are summarized in Table 1. For the larger values of $|dm_s/dz|$, it is evident that the maximum bottom temperature may be increased by increasing the heat flux and the salt gradient. For the smaller values of $|dm_s/dz|$, however, the maximum bottom temperature does not increase with increasing heat flux. For weakly stratified systems, the effect which increasing q_b has on reducing t_s limits the achievement of higher temperatures in the bottom mixed layer. If the objective is to maximimize temperatures associated with the bottom mixed layer, there is therefore an obvious trade-off between the need to maintain a high degree of stability, while operating with a large bottom heat flux.

Table 1. Maximum bottom layer temperature and number of days of stable operation, T_b(°C)/t_s(days), as a function of bottom heat flux and salt gradient

dm_s/dz (%/m)	$q_{b,c}$ (W m^{-2}) 80	100	120
−6.5	45/10	45/8	45/4
−13.0	74/33	75/22	67/14
−19.5	80/40+	87/36	93/27
−26.0	85/40+	94/40+	108/31

SUMMARY

A mathematical model has been developed for predicting the response of a salt-stratified solution to heating from below. In the time domain $t < t_p$, the model involves a bottom mixed layer and an overlying diffusive region; while for $t \geq t_p$, it involves top and bottom mixed layers and an intermediate diffusive region. The model is based on application of energy and salt balances to the mixed layers, use of an integral form of the heat equation for the diffusive region, and the use of established correlations for predicting the entrainment of stratified fluid and destabilization of the diffusive region. Comparisons between model predictions and laboratory experiments have been made in terms of the mixing layer temperatures, salt concentrations and heights, as well as in terms of the time required for thermal effects to penetrate to the top of the system and the time required to destabilize the diffusive region. All major data trends have been predicted by the model, and with the exception of the thermal penetration time, there is reasonable agreement between numerical results. Significant discrepancies in the thermal penetration time are attributed to limitations inherent in use of the integral form of the heat equation.

Parametric calculations have been performed to determine the effect of bottom heat flux and salt gradient on system response. The effect which increasing the heat flux has on reducing the time associated with stable operation of the diffusive region has been found to limit the extent to which the bottom mixed layer temperature can be increased by increasing the heat flux.

Acknowledgement—This work was supported by the National Science Foundation under Grant MEA-8009034.

REFERENCES

1. J. S. Turner, The behavior of a stable salinity gradient heated from below, *J. Fluid Mech.* **33**, 183–200 (1968).
2. C. J. Poplawsky, F. P. Incropera and R. Viskanta, Mixed layer development in a double-diffusive, thermohaline system, *J. Sol. Energy Engng* **103**, 351–359 (1981).
3. H. E. Huppert and P. F. Linden, On heating a stable salinity gradient from below, *J. Fluid Mech.* **95**, 431–464 (1979).
4. C. E. Nielson and A. Rabl, Salt requirement and stability of solar ponds, *Proc. Joint Conf. American Section Solar*

Energy Soc. and Solar Energy Soc. Canada, Vol. 5, pp. 183–187 (1976).

5. K. A. Meyer, A one-dimensional model of the dynamic layer behavior in a salt gradient solar pond, *Proc. Conf. American Section Int. Solar Energy Soc.*, pp. 763–767 (1981).
6. J. S. Turner, The coupled turbulent transports of salt and heat across a sharp density interface, *Int. J. Heat Mass Transfer* **8**, 759–767 (1965).
7. W. T. Lewis, F. P. Incropera and R. Viskanta, Interferometric study of stable salinity gradients heated from below or cooled from above, *J. Fluid Mech.* **116**, 411–430 (1982).
8. T. R. Goodman, Application of integral methods to transient nonlinear heat transfer, in *Advances in Heat Transfer*, Vol. 1, pp. 51–122 (1964).
9. J. Deardorff, G. Willis and B. Stockton, Laboratory studies of the entrainment zone of a convectively mixed layer, *J. Fluid Mech.* **100**, 41–64 (1980).
10. T. L. Bergman, The behavior of a salt-stratified, double-diffusive, thermohaline solution heated from below, M.S. Thesis, Purdue University, W. Lafayette, Indiana (1981).
11. P. Henrici, *Discrete Variable Methods in Ordinary Differential Equations*. John Wiley, New York (1962).
12. C. F. Gerald, *Applied Numerical Analysis*. Addison Wesley, Reading, U.S.A. (1980).
13. United States Office of Saline Water, Technical Data Book (1964).
14. H. Weinberger, The physics of the solar pond, *Sol. Energy* **8**, 45–56 (1964).
15. E. R. G. Eckert and R. M. Drake, *Analysis of Heat and Mass Transfer*. McGraw–Hill, New York (1972).

UN MODELE MULTI-COUCHES POUR LE DEVELOPPEMENT DE LA COUCHE DE MELANGE DANS UN SYSTEME THERMOHALINE CHAUFFE PAR LE BAS

Résumé—Un modèle mathématique est développé pour déterminer la réponse dynamique de la solution stratifiée de sel chauffée par le bas et les résultats sont comparés avec l'expérience. Tous les principaux comportements sont corroborés et un bon accord numérique est obtenu pour la température, la concentration de sel et l'épaisseur de la couche mélangée qui se forme en réponse au chauffage. La sousestimation du temps nécessaire pour que les effets thermiques pénétrent jusqu'au sommet de la solution est attribuée aux limitations associées à l'utilisation de l'équation intégrale d'énergie pour décrire les conditions dans la région stable. Des calculs paramétriques sont conduits pour déterminer l'effet du flux thermique à la base et du gradient stabilisant sur la réponse du système.

EIN MEHRSCHICHTEN-MODELL FÜR DIE ENTWICKLUNG DER MISCHZONE IN EINEM VON UNTEN BEHEIZTEN THERMOHALINEN DOPPELDIFFUSIONS-SYSTEM

Zusammenfassung—Es wurde ein mathematisches Modell entwickelt, um die dynamische Reaktion einer geschichteten Salzlösung auf Beheizung von unten zu bestimmen, und die Ergebnisse wurden mit den experimentellen Ergebnissen verglichen. Alle wesentlichen Tendenzen der Messungen wurden bestätigt und gute numerische Übereinstimmung für Temperatur, Salzkonzentration und Dicke der Bodenmischzone erhalten, welche sich als Folge der Beheizung bildet. Der Grund, daß sich zu geringe Werte für die Zeit ergeben, welche benötigt wird, bis die thermischen Effekte zum oberen Teil der Lösung vordringen, wird in der begrenzten Genauigkeit des Rechenverfahrens gesehen, bei dem die Zustände im stabilen Gebiet mit Hilfe der integralen Energiegleichung beschrieben werden. Es wurde eine Parameterstudie durchgeführt, um die Auswirkung des Wärmestroms am Boden und des stabilisierenden Konzentrationsgradienten des Salzes auf die Reaktion des Systems zu bestimmen.

МНОГОСЛОЙНАЯ МОДЕЛЬ РАЗВИТИЯ НЕИЗОТЕРМИЧЕСКОГО СЛОЯ СМЕШЕНИЯ ПРИ КОНЦЕНТРАЦИОННОЙ СТРАТИФИКАЦИИ

Аннотация—Разработана математическая модель для определения влияния нагрева снизу на динамнку стратифицированного солевого раствора. Результата расчетов сравниваются с экспериментальными данными. Подтверждены известные закономерности и получено хорошее численное согласие для температуры, концентрации соли и толщины нижнего слоя смешения, образующегося при нагревании. Заниженное значение времени, требуемого для распространения тепловых возмущений в верхнюю область смешения, объясняется ограничениями, налагаемыми при использовании интегрального уравнения энергии для описания процесса переноса в области устойчивой стратификации. Выполнены параметрические расчеты для определения влияния подвода тепла снизу и устойчивого градиента солености на поведение системы.